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COMMENT

COMMENT ON 'ON THE USE OF THE REACH-BACK CHARACTERISTICS METHOD FOR CALCULATION OF DISPERSION'[1]

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The authors present a modification of the two-point fourth-order method of Holly and Preissmann[2] (HP method) which they call the reach-back method (HPRB method). The purpose of this note is to clarify the nature of the HPRB method and its relation to the HP method and to comment on the reason for the reported improvement of the error characteristics of the HPRB method compared to the HP method.

The difference between the HP method and the HPRB method is that the HPRB method projects a characteristic back $m \geqslant 1$ time levels whereas in the HP method the characteristic is projected back one time level. When $m=1$, the methods are identical. In Figure 1 the paths of a set of characteristics for the $m=2$ case are illustrated. From the figure it is evident that two independent solutions are being computed, one for the solid line and one for the heavy dashed line. Each set of solutions is identical to an HP solution with the time step increment set at $2\Delta t$. Consequently, the HPRB method of reach-back m is identical to a set of m HP solutions with time step $m\Delta t$ that have staggered starting times. The solution at the final time step of the HPRB method with reach-back m is identical to that of the HP method with time step $m\Delta t$; however, the number of intermediate HPRB solutions will be m times the number of intermediate HP solutions.

The HP method belongs to the Eulerian–Lagrangian class of methods (ELMs). The author's error analysis and results are based on exact tracking and constant velocity. For the pure advection problem with exact tracking, the error in the solution of ELMs is due to the interpolation required at the feet of the characteristics. The ELM interpolation error in a given time step is partially dependent on the location of the foot of the characteristic in relation to the nodes of the element.[3,4] For linear test problems with constant velocity and constant time step interval the position of the foot of the characteristic relative to the nodes is the same for every time step and, as noted by the authors, the relative position is strictly a function of the decimal portion of the Courant number. However, in engineering applications flow lines are curved and velocities vary across the domain, so in general the location of the feet of the characteristics will vary relative to the nodes and the variation in interpolation error per time step will tend to average out. Consequently the interpolation error per time step in practical applications will be more or less constant.

On the other hand, the total interpolation error will increase with the number of time steps taken.[3,4] Assuming that tracking is exact and tracking error is independent of the time step size, larger time steps will mean that fewer time steps are required to reach the final simulation time and less interpolation error is accumulated. Hence, for the HPRB method or the HP method with an equivalently large time step the reduced number of time steps leads to a reduction in the

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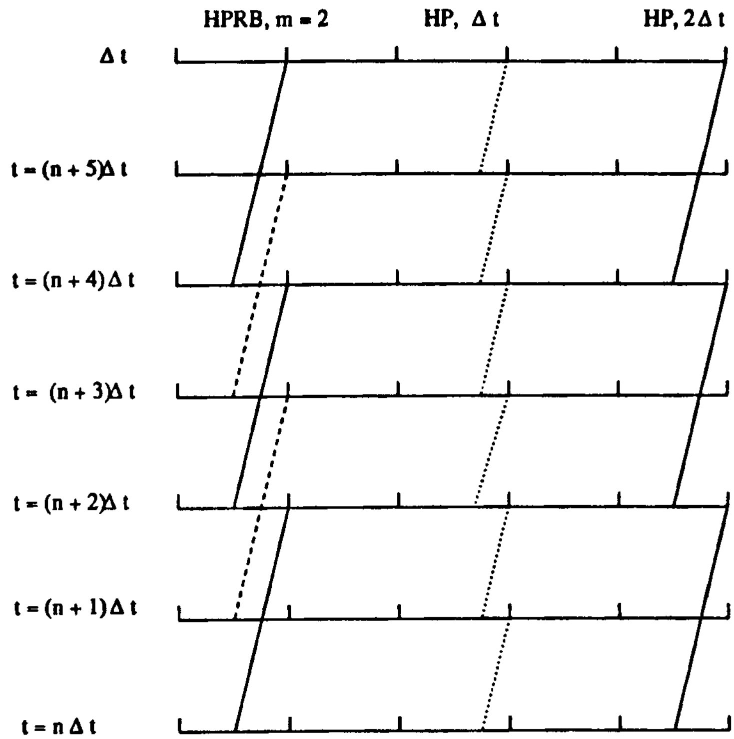

Figure 1. Characteristic paths. Characteristics are projected back from selected nodes at a series of time steps. Where the HP method tracks back one time level to find the interpolation position, the HPRB method with $m=2$ follows the characteristic back two time levels. When $m=2$, the HPRB method is actually generating two independent solutions, and the HP solution with $2\Delta t$ time step is identical to one of the solutions

accumulated interpolation error and a reduction in the numerical damping that is caused by interpolation error. When dispersion is present, error is introduced owing to the time truncation error associated with the dispersion term. As the size of the time step increases, the time truncation error associated with the dispersion term increases and the accumulation of interpolation error from the tracking step decreases. Consequently, when dispersion is present, an optimal time step size exists,[3] leading to an optimal reach-back parameter m. The authors' observation that the solution will always improve as m increases is only true when no dispersion is present.

In summary, the HPRB method of reach-back $m > 1$ has less numerical damping than the HP method when both use time step Δt because of the larger effective time step size of the HPRB method. The final solutions of the HPRB method and the HP method with the equivalent time step size are identical. The difference between the HPRB method and the HP method with time step $m\Delta t$ is that the HPRB method generates more intermediate solutions at the cost of more computational effort.

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